ASTRONOMICAL REFRACTION AND THE EQUINOX SUNRISE

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Abstract. A ray-tracing model is used to illustrate the influence of astronomical refraction on the azimuthal position of the equinox sunrise. The variation in sunrise azimuth as a function of the range in astronomical refraction and the observer's latitude is also investigated.

Résumé. Un modèle basé sur la technique de lancer de rayons est utilisé pour illustrer l'influence de la réfraction astronomique de la position de l'azimut à la levée du soleil durant l'équinoxe. La variation de l'azimut à la levée du soleil en fonction du champ de la réfraction astronomique et de la latitude de l'observateur est aussi examinée. SEM

1. Introduction

The original motivation for this paper was the *Journal* article by Attas & McMurry (1999) entitled "Nailing the Equinox Sunrise." In the article it is stated that "on the equinox, the Sun should rise due east." Since the Sun crosses the celestial equator at the equinox, and since the celestial equator crosses the horizon at 90° and 180° azimuth (due east and west), that statement appears on the surface to be correct. Once the effects of astronomical refraction are considered, however, the phenomenon becomes a little more complex.

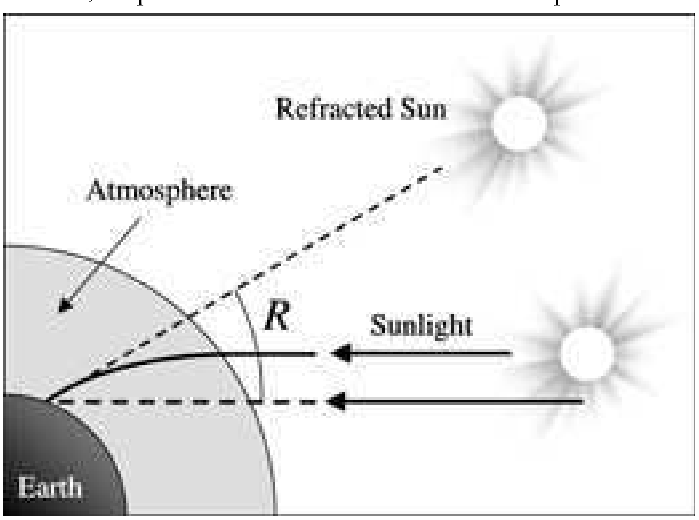

Fig. 1 — A schematic of astronomical refraction. The angle *R* is the amount of refraction.

Before proceeding, a formal definition of sunrise needs to be established. Most sources define the moment of sunrise and sunset as the time when the upper limb of the Sun makes contact with a horizon (Green 1985). In other words, sunrise and sunset occur when the upper limb of the Sun reaches an altitude of 0°.

Astronomical refraction is the bending of light from celestial objects by the Earth's atmosphere. The overall effect is to increase the observed altitude of a celestial object as it gets closer to the horizon (see figure 1). Often empirical formulae or tables are used to estimate the amount of astronomical refraction at a particular altitude above the horizon (*e.g.* Orlov 1956; Green 1985; Seidelmann 1992). Such tables and formulae become highly inaccurate very near the horizon. That is a consequence of the variability of atmospheric properties, which change with geography, season, time of day, and the presence of passing air masses. In Sampson (2000) it is shown that a ray-tracing model employing fine-scale atmospheric sounding profiles appears to be fairly successful at modeling astronomical refraction near the horizon.

According to Snell's Law, the angle of refraction of a light ray of a given wavelength passing between two transparent media is a function of the refractive indices of the two media. In turn, the refractive index is a function of the density of the medium. Since the atmosphere is compressible, it has a continuous density gradient. The density of the atmosphere at any particular location is dependent on its temperature, pressure, and composition. The highest variability in atmospheric composition is attributable to water vapour. Since the temperature, pressure, and water vapour content vary both spatially and temporally, it is not surprising that astronomical refraction can also change with the atmospheric conditions.

In this paper the unrefracted Sun is referred to as the geometric Sun. From the definition of sunrise and sunset, the azimuthal location of the geometric equinox sunrise can be shown to be less than 90° (north of due east in the northern hemisphere). The magnitude of the difference depends on the observer's latitude and the apparent diameter of the Sun (see figure 2). From figure 2 it is apparent that the only location where the equinox sunrise occurs exactly at the east point is on the equator.

2. The Refraction Model

The model presented here is a time-reversed ray-tracing model that uses an incremental search routine. The rays are sent out from the observer instead of from the Sun. The path of such time-reversed light rays is exactly the same as for time-forward rays. A planetary

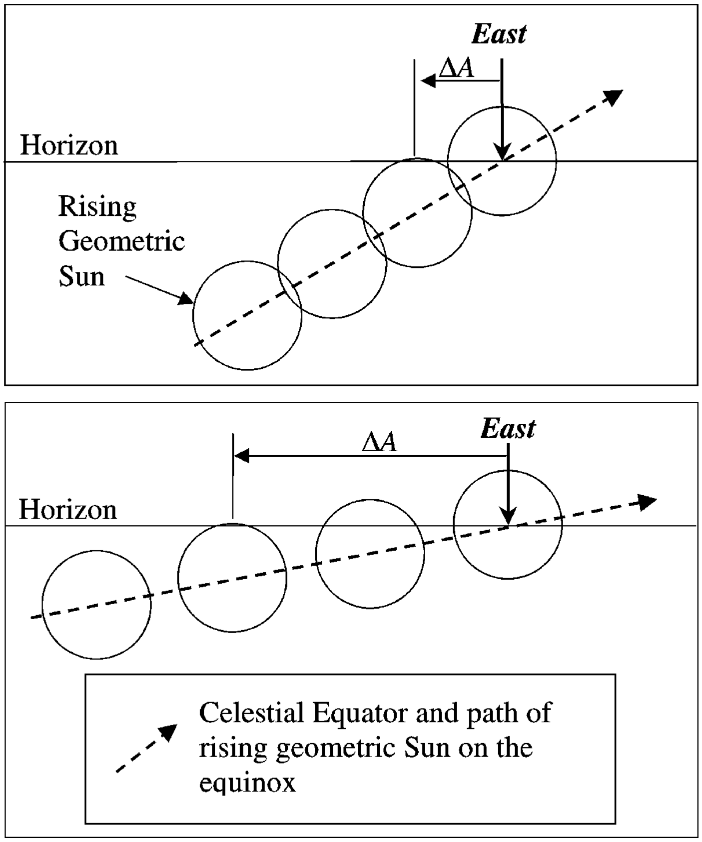

Fig. 2 — The dependence of the geometric sunrise azimuth on the latitude of the observer. The upper schematic is for a lower latitude observer, while the upper diagram is for an observer nearer the pole. The angle ΔA is the difference in azimuth between the sunrise point and due east. On the equinox the centre of the geometric Sun crosses the horizon at an azimuth of 90° (due east), as long as the time of sunrise is the same as the time of the equinox. On the equator the equinox sunrise occurs due east.

orbital model first calculates the celestial co-ordinates of the geometric Sun (Meeus 1988). From a first guess for the amount of astronomical refraction computed from an empirical formula (Seidelmann 1992), the software computes an initial estimate for the ray angle. A ray of particular wavelength is then propagated from the observer towards the Sun. Once the ray exits the atmosphere, its miss-angle is calculated. The initial angle is then adjusted and another ray is propagated until the miss-angle is 6 arcseconds or less. That level of tolerance was chosen to match the accuracy of current experiments into photogrammetric measurements of low altitude solar images.

As a ray propagates through the atmosphere, its trajectory is advanced by increments of 0.36 arcsecond with respect to the centre of the Earth. That translates into a horizontal distance of about 11 metres on the Earth's surface. At the end of each increment the refractive index is computed according to the atmospheric conditions at that location. Snell's Law and a curvature term (Bruton 1996) are then applied to the resulting incident ray. The model assumes a horizontally homogeneous atmosphere, since the vertical density gradient is much larger and, therefore, has far more influence on astronomical refraction.

The vertical profile of the atmosphere is found from weather balloon data. They are obtained from the twice-daily rawindsondes launched from the Stony Plain Environmental Monitoring Station about 25 km west of Edmonton. By convention, all Canadian rawindsondes are launched at 23:15 and 11:15 UTC, fortuitously timed for sunrises and sunsets from Edmonton. The current rawindsonde model is the Vaisala RS 80, which provides temperature, pressure, and humidity every 10 seconds (approximately every 50 metres). The temperature, pressure, and humidity for each ray increment are interpolated from the measurements. The refractive index is then computed using a scheme developed by Ciddor (1996).

3. Simulating the Autumnal Equinox Sunrise

In order to eliminate the effects of solar motion along the ecliptic from the azimuthal location of the autumnal sunrise, the solar declination is set at zero. In 1999 the moment of the autumnal equinox and the sunrise are approximately concurrent at a longitude of 86° W. A latitude of 50° N was chosen for the simulation, since Attas and McMurry were located at that latitude when their photographs were taken. An 11:15 UTC (05:15 MDT) rawindsonde launch from September 22, 1997, was chosen from the library of atmospheric soundings from the Stony Plain Alberta station. A sunrise was observed from Edmonton on that day. Climatic conditions between southern Manitoba and central Alberta are considered to be quite similar. The differences in elevation and meteorological conditions between the two locations, however, make the profile only roughly applicable to the September 23, 1998, situation.

The model was run until a small portion of the yellow light (580.0 nm) image of the Sun appeared above the horizon. The results appear in figure 3. The simulation suggests that autumnal equinox sunrise occurred at an azimuth of 88° 53´ — over a full degree north of due east. It is also apparent from the figure that equinox sunrise is about 50´ further north than the geometric sunrise.

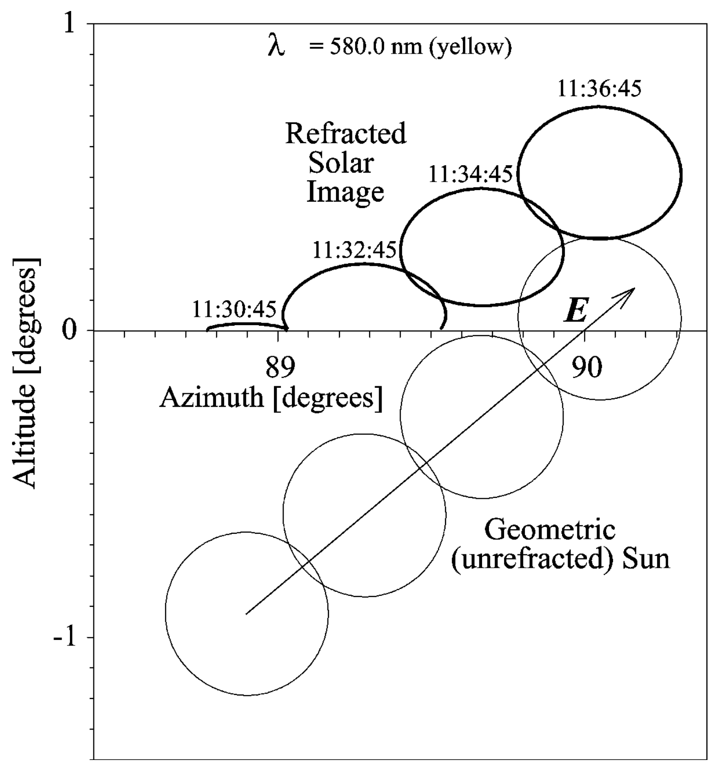

Fig. 3 — Simulated 1999 autumnal equinox sunrise from 89°W longitude and 50°N latitude. The wavelength of light is 580.0 nm (yellow), and times are in UTC.

4. Latitude, Refraction, and the Equinox Sunrise

As illustrated in figure 2, the azimuthal location of sunrise is also dependent on the observer's latitude. The azimuth A of the sunrise or sunset point can be calculated from the cosine law for spherical trigonometry (Green 1985):

$$\cos A = \frac{\sin\delta - \sin\phi \sin a}{\cos\phi \cos a} \qquad (1)$$

where δ is the declination of the Sun, ϕ is the latitude of the observer, and a is the altitude of the centre of the Sun at the moment of sunrise or sunset. The altitude of the centre of the Sun at sunrise or sunset is the sum of the solar semi-diameter and the amount of astronomical refraction. At the time of the 1999 autumnal equinox, the solar semi-diameter was 15′ 56″.4. The amount of refraction at sunrise or sunset is highly variable, and can range from about 30′ under normal conditions to more than 4° when a Novaya Zemlya arctic mirage occurs (Lehn 1974). At the moment of the equinox, the declination of the Sun is equal to zero. A plot of the variation of sunrise azimuth versus latitude can be seen in figures 4 and 5. From the graphs it is apparent that the change in the azimuth of the sunrise is greatest near the pole. It is also obvious that the azimuthal location of sunrise is highly dependent on the amount of astronomical refraction.

Figures 4 and 5 also show that the higher the latitude, the more sensitive the azimuth of the equinox sunrise is to changes in astronomical refraction. Figures 6 and 7 show a plot of latitude versus the difference in equinox sunrise azimuth between 30′ and 1° astronomical refraction.

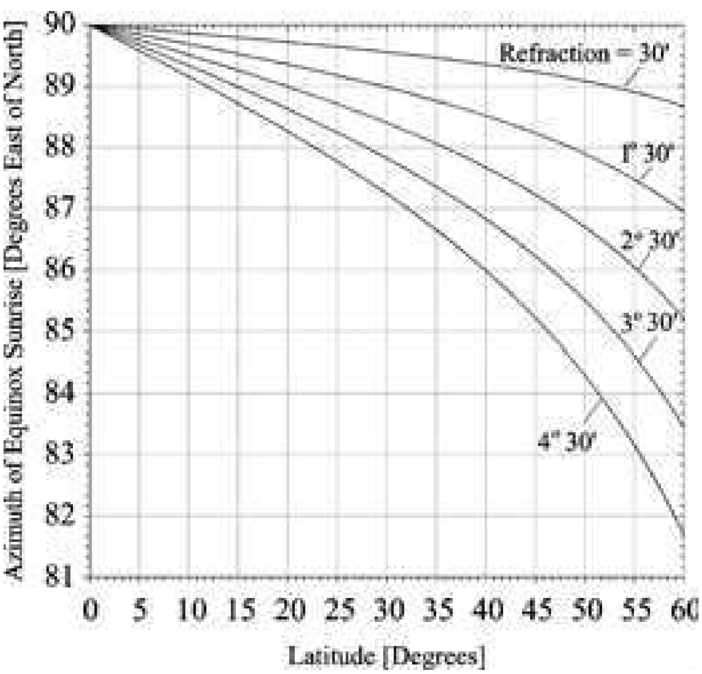

Fig. 4 — The relationship between sunrise azimuth, latitude, and astronomical refraction. Under more normal circumstances the amount of astronomical refraction is between 30′ and 48′ (Sampson 1994). Extreme refraction events appear to be confined to the polar regions (Lehn 1974), although sunrise events with over 1° of refraction have been recorded by the author during summer sunrises from Edmonton (and over 2° for winter sunrises).

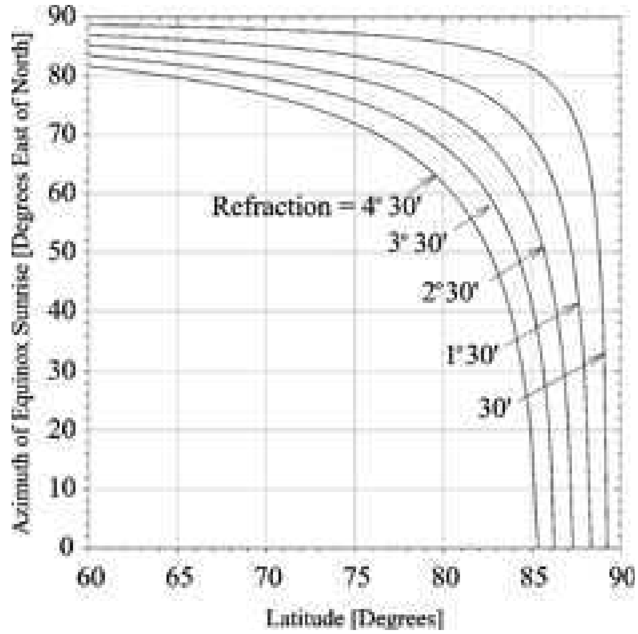

Fig. 5 — See figure 4.

5. Conclusion

On the surface, the exact azimuthal location of the sunrise and sunset point appears to be determined solely by celestial and geographic co-ordinates. As the preceding arguments have demonstrated, however, the azimuthal location of the sunrise point is also a function of astronomical refraction. Since astronomical refraction can vary with the conditions of the atmosphere, the accurate location of the sunrise point is therefore difficult to forecast without a detailed understanding of the atmosphere at the time of the event.

Astronomical refraction decreases the azimuth of the sunrise point. From figures 4, 5, 6, and 7 it is apparent that the azimuthal

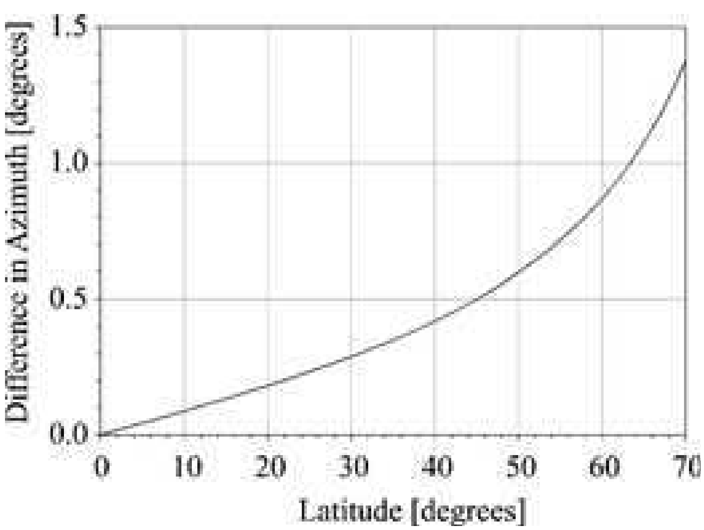

Fig. 6 — The difference between the equinox sunrise azimuth produced by 30′ of astronomical refraction and that produced by 1° of astronomical refraction plotted as a function of latitude. This illustrates the increase in sensitivity of the sunrise azimuth to variations in astronomical refraction as the observer approaches the poles.

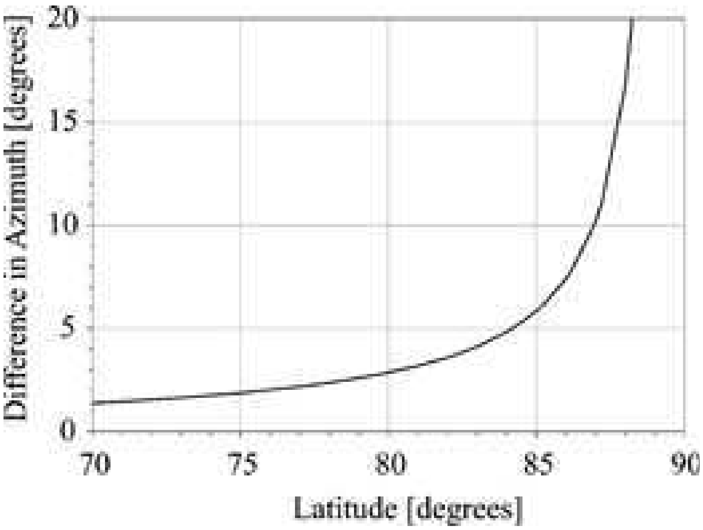

Fig. 7 — See figure 6.

location of the sunrise (or sunset) point becomes more sensitive to changes in astronomical refraction as latitude increases. It also appears that such northerly regions are more likely to experience extreme refraction events (Lehn 1974), which further enhance the variability in sunrise and sunset azimuth.

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References

Attas, M. & McMurry, J. 1999, JRASC, 93, 163

Bruton, D. 1996, Ph.D. Thesis, Texas A&M University

Ciddor, P. E. 1996, Applied Optics, 35, 1566

Green, R. M. 1985, Spherical Astronomy (Cambridge University Press: Cambridge), p. 520

Lehn, W. H. 1974. Journal of the Optical Society of America, 69, 776

Meeus, J. 1988, Astronomical Formulae for Calculators (Willmann-Bell: Richmond, Virginia), p. 218

Orlov, B. A. 1956, Refraction Tables of the Pulkova Observatory, 4th Edition, (Russian Academy of Science Press: Leningrad, Moscow)

Sampson, R. D. 1994, M.Sc. Thesis, University of Alberta

Sampson, R. D. 2000, Ph.D. Thesis, University of Alberta, in preparation

Seidelmann, K. P. 1992, Explanatory Supplement to the Astronomical Almanac (University Science Books: Mill Valley, California), p. 752

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